Properties of the interface generated by the collision of two growing interfaces

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The collision interface between two Eden clusters is studied in two dimensions. Relevant exponents characteristic of the interface collision are evaluated. The roughness exponent of the collision interface is the same than that of a single Eden interface. The dynamic of the collision interface departs from standard scaling, but obey simple rescaling. [S1063-651X(97)12411-4]

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The existence of an evolving interface is a relevant characteristic present in a wide variety of physical, chemical and biological systems and processes. Among others, one should mentioned aggregation [1], film growth by vapor deposition, chemical deposition and molecular-beam epitaxy [2], propagation of forest fires [3], solidification [4], bacterial growth [5], propagation of diffusion fronts [6], propagation of reactant's fronts in catalyzed reactions [7], etc.

The scaling theory describing the dynamic evolution of a self-affine interface is very well established [2,8,9]. In fact, let $h(\vec{r},t)$ be the height of the aggregate at position $\vec{r}$ and time t. If the mean height is $\langle h(t)\rangle$, then the long-wavelength fluctuations of the interface width $[w(t)]$ are given by

$$w(t)=\langle[h(t)-\langle h(\vec{r},t)\rangle]\rangle^{1/2}. \quad (1)$$

If the aggregate is grown in a finite lattice of lineal dimension L, the interface width becomes $w(L,t)$, and scales as [10]

$$w(L,t)\propto L^{\alpha}F(t/L^{z}), \quad (2)$$

where $F(x)\propto x^{\beta}$ for $x\ll 1$ and $F(x)\rightarrow 1$ for $x\gg 1$, with $z=\alpha/\beta$. Thus for a finite sample and $t\rightarrow\infty$, one has $w(L)\propto L^{\alpha}$; that is, the width of the growing interface reaches a statistically stationary state.

The aim of this work is to study the properties of the interface generated upon the collision of two interfaces. This

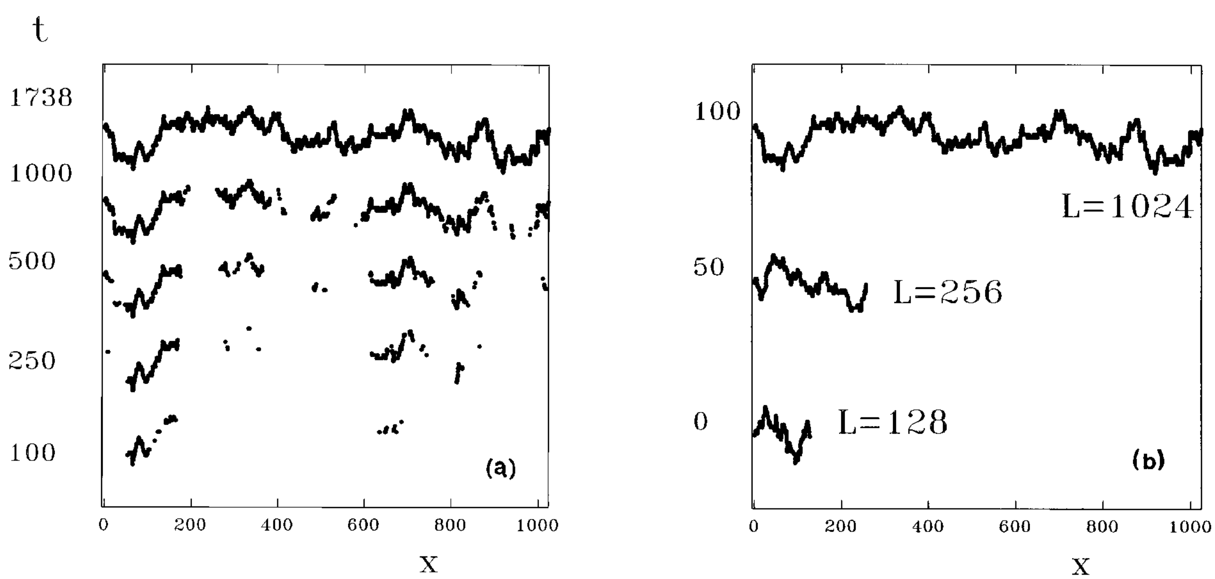

FIG. 1. Snapshot configurations of the collision interface. The horizontal axis shows the coordinates of the collision sites measured in lattice units. (a) Time evolution of a collision interface in a lattice of side $L=1024$. The vertical axis indicates the Monte Carlo time, measured in MCS's, at which each snapshot has been obtained. (b) Collision interfaces obtained using lattices of different width as shown in the figure. The origin of the vertical axis, measured in lattice units, is taken to be arbitrary, so that the average interface positions $\langle h\rangle$ corresponding to different snapshots are separated by 50 lattice units.

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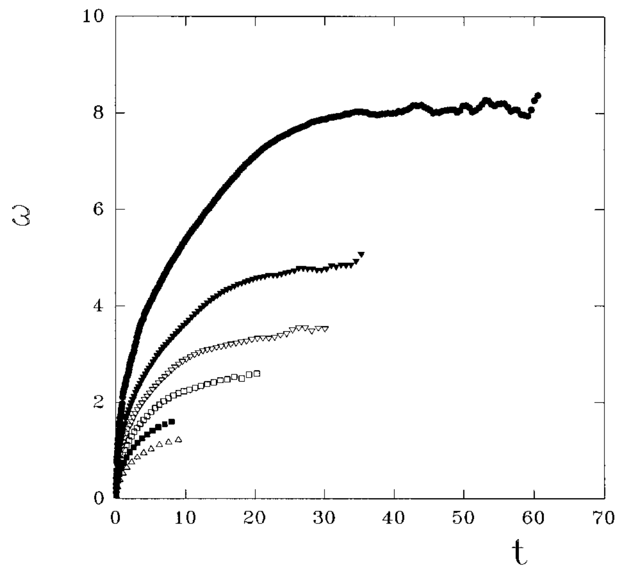

FIG. 2. Plots of w; measured in lattice units; vs t; measured in MCS's; for collision interfaces obtained in lattices of different width. From top to bottom: $L=1024$, 256, 128, 64, 32, and 16. Data are evaluated averaging over 10^4 collision interfaces.

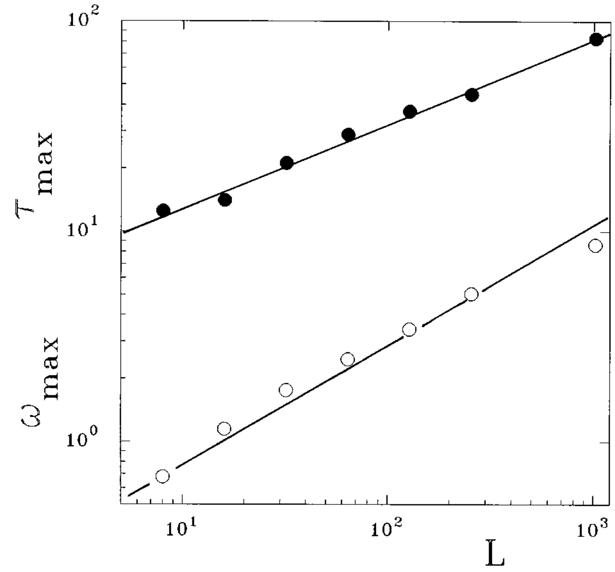

FIG. 3. Log-log plots of τ_{max}, measured in MCS's; and w_{max}; measured in lattice units; vs L; measured in lattice units; for Eden collision interfaces. Data are evaluated averaging over 10^4 collision interfaces.

study is motivated by various physical realizations, e.g., due to the collision of fire fronts in burning experiments [11] or forest-fire evolution [3], as a result of interference between chemical waves [12], upon solidification of inmiscible fluids, etc.

Specifically, this work reports a study of the collision interface due to two Eden clusters [13] on the square lattice in two dimensions. Two Eden clusters are grown starting from the opposite sides of a rectangular sample of lineal dimensions $L\times M$ with $L\ll M$. Both clusters grow independently until their respective interfaces reach the stationary state, such as $w(L,t\rightarrow\infty)\propto L^{\alpha}$; see Eq. (2). So our interest is focused on the properties of the collision interface formed after saturation of the long-wavelength fluctuations of the interfaces of the individual Eden clusters.

Eden cluster growth proceeds according to standard algorithms keeping a list of growing centers. A collision event takes place just when the last occupied growing center of one cluster has at least one occupied neighbor belonging to the other cluster. Then a collision interface site is identified at the position between both occupied sites of different clusters. One Monte Carlo time step (MCS) involves the incorporation of L particles randomly distributed between the growing centers of both Eden clusters. The dynamic evolution of the collision interface if followed after the occurrence of the first collision event at $t=0$. The collision simulation stops when growing centers of both clusters become exhausted, so the lifetime of the interface collision (τ) is the time elapsed between the first and last collision events. It should be noted that the study of a collision interface essentially differs from the investigation of the interface formed during the competitive growth of clusters, as studied by Derrida and Dickman [14].

Figure 1(a) shows snapshots of the collision interface obtained at different times for a collision on a lattice of width $L=1024$. From these snapshots, it follows that early collision points are not randomly distributed throughout the sample, but instead that collisions events occur simultaneously at various and well-localized zones of the collision interface. After these early collision events, the incipient collision zones spread sidewise, forming the entire interface. On the other hand, Fig. 1(b) shows snapshots of the final collision interface obtained using lattices of different width. Here the self-affinity of the collision interface can be qualitatively observed.

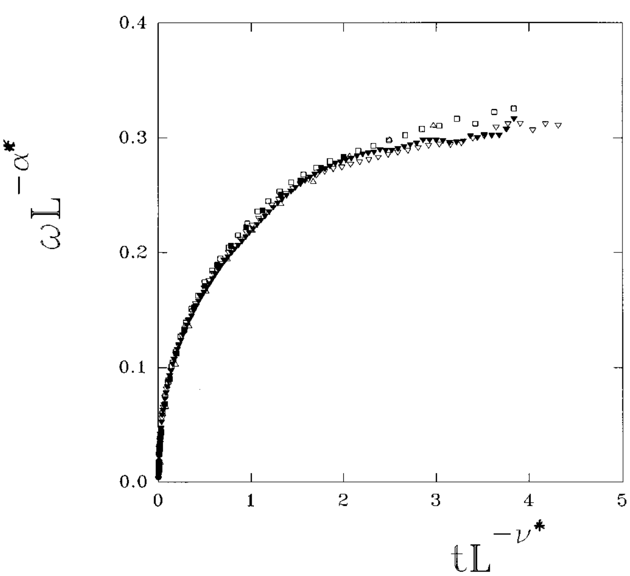

FIG. 4. Rescaled plots of $w(L,t)/L^{\alpha^*}$ vs t/L^{ν^*} for the data shown in Fig. 2.

Figure 2 shows the time evolution of the interface width obtained using samples of different width. It follows, from Fig. 2, that both the maximum lifetime τ_{max} of the collision and the maximum width w_{max} depend on the lattice width L. So, Fig. 3 shows logarithmic-logarithmic plots of τ_{max} and w_{max} versus L. The obtained straight lines suggest a power-law dependence of the forms

$$w_{max}(L) \propto L^{\alpha^*} \tag{3}$$

and

$$\tau_{max}(L) \propto L^{\nu^*}, \tag{4}$$

where α^* and ν^* are exponents characteristic of the interface collision. Least-square fits of the data yield $\alpha^* \cong 0.51 \pm 0.02$ and $\nu^* \cong 0.40 \pm 0.02$. This result is consistent with $\alpha = \alpha^* = \frac{1}{2}$, so the roughness exponent of the interface collision should be the same as that of the individual interfaces of the Eden clusters prior to collision.

The obtained results also suggest that, after a simple rescaling of the axis of Fig. 2, one should obtain data collapsing, as, e.g., is shown in Fig. 4, where a plot of $w(L,t)/L^{\alpha^*}$ versus t/L^{ν^*} is presented. It should be noted that the simple rescaling observed is not compatible with the standard scaling approach given by Eq. (2).

Summing up, the properties of the collision interface between two Eden clusters have been investigated, and relevant exponents of the dynamic collision process have been evaluated. While the roughness exponent of the collision interface seems to be the same as that of the Eden interface, the dynamic of the collision interface width departs from the standard scaling law and obeys simple rescaling. It is expected that the obtained results would stimulate experimental evaluations of the collision interface exponents which may be accessible in many physical, chemical, and biological realizations.

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